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1995 J. Phys. A: Math. Gen. 28 L239

(http://iopscience.iop.org/0305-4470/28/8/003)

J. Phys. A: Math. Gen. 28 (1995) L239–L244. Printed in the UK

LETTER TO THE EDITOR

# Identical particles and permutation group

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Received 17 February 1995

**Abstract.** Second quantization is studied. Creation and annihilation operators are shown to be related on the same basis to both the algebra $h(1)$ and to the superalgebra $osp(1|2)$ which are shown to be compatible with both Bose and Fermi statistics. The two algebras are completely equivalent in the one-mode sector but, because of the grading of $osp(1|2)$, differ in the many-particle case. The possibility of an unorthodox quantum field theory is suggested.

Claiming that a permutation of two particles has been performed requires that the particles themselves can be distinguished. An idealized operational procedure to this effect would be as follows. One first attaches a label to each particle (i.e. a quantum number identifying its state) in order to distinguish it from any other, then one interchanges the particles and, finally, one looks once more at the labels to make sure that the exchange has been properly performed. However, one of the fundamental hypotheses of quantum field theory is that particles should be treated as identical and indistinguishable; for this reason the permutation group is not truly related *ab initio* to second quantization but is introduced into the theory only at a second stage when the $n$-particle states are described in terms of first quantization observables.

This has the consequence that the usual connection between the algebraic properties of second quantization operators and the statistics of the particles turns out to possess some arbitrariness. In order to prove this statement, we shall build explicitly, in terms of *anticommuting* creation and annihilation operators, a new scheme where, by imposing the symmetry or antisymmetry of the particle states, both bosons and fermions can be simultaneously constructed. As briefly discussed at the end of this letter, the construction presented should be considered as an example of a much more general and far-reaching feature: since there is no necessary connection between the observables over the Fock space and the particle statistics, we are allowed not only to relate *both* fermions and bosons to the Weyl–Heisenberg algebra $h(1)$, but the scheme is also extendable to more complex relations among observables (e.g. quantum algebras) and/or exotic statistics (e.g. anyons). All these structures are, indeed, compatible. The one exception is the standard structure for fermions (provided by the superalgebra $h(1|1)$) which is consistent with fermions only. It should be mentioned that the approach presented in this letter was inspired by the property that the algebraic structures relevant to second quantization physics are Hopf algebras [1]: this is just the coproduct (i.e. the multi-mode description), trivial for Lie algebras and brought to our attention as a result of studies of quantum algebras, which is the basis for our construction and dramatically discriminates between the different descriptions.

More formally, let us begin by showing how the creation and annihilation operators can be related, on the same basis to both the algebra $h(1)$ and to the *superalgebra* $osp(1|2)$. $h(1)$ is customarily defined [2] to be generated by the four operators $(a, a^\dagger, \mathbf{1}, N)$, with commutation relations

$$[a, a^\dagger] = 1 \qquad [N, a] = -a \qquad [N, a^\dagger] = a^\dagger \qquad [\mathbf{1}, \bullet] = 0. \tag{1}$$

Upon characterizing the unitary representations (i.e. those for which $N^\dagger = N$, $(a^\dagger)^\dagger = a$) with the spectrum of $N$ bounded below by their lowest eigenvalue $n_0$, one can write

$$\begin{aligned} a^\dagger|k+n_0\rangle &= \sqrt{k+1}\,|k+n_0+1\rangle \\ a|k+n_0\rangle &= \sqrt{k}\,|k+n_0-1\rangle \\ N|k+n_0\rangle &= (k+n_0)|k+n_0\rangle \qquad k \in \mathbb{N}. \end{aligned}$$

The usual Fock space $\mathcal{F}$ is obtained for $n_0 = 0$, usually adopting the relation $N \equiv a^\dagger a$ (which is just one of the solutions of the equations $[N, a] = -a$, $[N, a^\dagger] = a^\dagger$):

$$\begin{aligned} a^\dagger|n\rangle &= \sqrt{n+1}\,|n+1\rangle \\ a|n\rangle &= \sqrt{n}\,|n-1\rangle \\ N|n\rangle &= n|n\rangle \qquad n \in \mathbb{N}. \end{aligned} \tag{2}$$

A related $\mathbb{Z}_2$-graded structure will be considered here, starting from the set of three operators $\mathcal{S} \equiv (a, a^\dagger, H)$ with $H$ even and $a$ and $a^\dagger$ odd. $\mathcal{S}$ is characterized *uniquely* by the relations

$$\{a, a^\dagger\} = 2H \qquad [H, a] = -a \qquad [H, a^\dagger] = a^\dagger \tag{3}$$

(i.e. $N$ as in (1) and $H$ is not assumed to be a function of $a$ and $a^\dagger$) and is a *subset*, not a *sub*-algebra, of the $\mathbb{Z}_2$-graded algebra $osp(1|2)$ [3]. In fact completion of $\mathcal{S}$ to the whole of $osp(1|2)$ requires the introduction of an additional set $\mathcal{S}' \equiv (J^-, J^+)$ in the even sector, such that

$$\{a^\dagger, a^\dagger\} = 2J^+ \qquad \{a, a\} = 2J^-. \tag{4}$$

Equations (3) and (4) imply the algebra closure:

$$\begin{aligned} &[J^+, a] = -2a^\dagger \qquad [J^-, a^\dagger] = 2a \qquad [J^+, a^\dagger] = 0 = [J^-, a] \\ &[J^+, J^-] = -4H \qquad [H, J^\pm] = \pm 2J^\pm. \end{aligned} \tag{5}$$

The bosonic sector $\mathcal{B} \equiv (J^-, J^+, \frac{1}{2}H)$ is isomorphic to $su(1,1)$ in the direct sum of the representations $\kappa = \frac{1}{4}$ and $\frac{3}{4}$ [4].

An explicit analysis shows that the set $\mathcal{S}$ with relations (3) is sufficient to give rise to unitary representations of $osp(1|2)$ that have the spectrum of $H$ bounded below, and can be characterized by the lowest non-negative eigenvalue of $H$, say $h_0$. Explicitly,

$$\begin{aligned} a^\dagger|h_0+2k+1\rangle &= \sqrt{2(k+1)}\,|h_0+2k+2\rangle \\ a^\dagger|h_0+2k\rangle &= \sqrt{2(k+h_0)}\,|h_0+2k+1\rangle \\ a|h_0+2k+1\rangle &= \sqrt{2(k+h_0)}\,|h_0+2k\rangle \\ a|h_0+2k\rangle &= \sqrt{2k}\,|h_0+2k-1\rangle \\ H|h_0+k\rangle &= (h_0+k)|h_0+k\rangle \qquad k \in \mathbb{N} \end{aligned} \tag{6}$$

where the partition of states in two classes exhibits the existence of supersymmetric doublets.

The main point of our derivation is the fact that equations (6), with $h_0 = \frac{1}{2}$, read

$$\begin{aligned} a^\dagger|h\rangle &= \sqrt{h+\tfrac{1}{2}}\,|h+1\rangle \\ a|h\rangle &= \sqrt{h-\tfrac{1}{2}}\,|h-1\rangle \\ H|h\rangle &= h|h\rangle \qquad h \in \mathbb{N}+\tfrac{1}{2} \end{aligned} \tag{7}$$

that is they coincide with equations (2) provided that the identification $h = n + \frac{1}{2}$ is implemented. This means that the closed subset $\mathcal{S}$ of $osp(1|2)$ defined by (3) and, by induction, the whole of $osp(1|2)$ shares the representation (2) with the Weyl–Heisenberg algebra $h(1)$. Thus, the Fock space $\mathcal{F}$ provides a faithful representation for both $h(1)$ (for $n_0 = 0$) and $osp(1|2)$ (for $h_0 = \frac{1}{2}$).

Second quantization is based essentially on the relations (2). We suggest that the creation and annihilation operators may, therefore, be interpreted as belonging either to $osp(1|2)$ or to $h(1)$. The key point in our argument is that if one considers the algebra as being generated by the defining commutation relations only, any physical interpretation is contained in equations (2) and it turns out to be, essentially, irrelevant whether one selects $osp(1|2)$ or $h(1)$. However, when one deals with many-particle states, the two schemes lead to self-consistent yet mutually unequivalent descriptions. The reason why this may happen is that $h(1)$ and $osp(1|2)$ are Hopf algebras (more precisely, $osp(1|2)$ is a *super* Hopf algebra). Any Hopf algebra, say $\mathcal{A}$, has, among its defining operations, the coproduct $\Delta : \mathcal{A} \to \mathcal{A} \otimes \mathcal{A}$ (in fact, in the representation considered here this is necessary in that it implies that the action of the algebra is well defined on $\mathcal{F} \otimes \mathcal{F}$ and, by induction, on $\mathcal{F}^{\otimes n}$). For both $h(1)$ and $osp(1|2)$ $\Delta$ is, of course, primitive.

In $h(1)$ one has

$$\begin{aligned} \Delta(a) &= \frac{1}{\sqrt{2}}\,(a \otimes \mathbf{1} + \mathbf{1} \otimes a) \equiv \frac{1}{\sqrt{2}}\,(a_1 + a_2) \\ \Delta(a^\dagger) &= \frac{1}{\sqrt{2}}\left(a^\dagger \otimes \mathbf{1} + \mathbf{1} \otimes a^\dagger\right) \equiv \frac{1}{\sqrt{2}}\left(a_1^\dagger + a_2^\dagger\right) \\ \Delta(N) &= N \otimes \mathbf{1} + \mathbf{1} \otimes N \equiv N_1 + N_2 \qquad \Delta(\mathbf{1}) = \mathbf{1} \otimes \mathbf{1}. \end{aligned} \tag{8}$$

The coalgebra for the superalgebra $osp(1|2)$ looks quite similar:

$$\begin{aligned} \Delta(a) &= a \otimes \mathbf{1} + \mathbf{1} \otimes a \equiv a_1 + a_2 \\ \Delta(a^\dagger) &= a^\dagger \otimes \mathbf{1} + \mathbf{1} \otimes a^\dagger \equiv a_1^\dagger + a_2^\dagger \\ \Delta(H) &= H \otimes \mathbf{1} + \mathbf{1} \otimes H \equiv H_1 + H_2. \end{aligned} \tag{9}$$

However, since $a$ and $a^\dagger$ are odd, whereas $H$ is even, we have for $c, d, e, f \in osp(1|2)$, the multiplication law on $\mathcal{F} \otimes \mathcal{F}$:

$$(c \otimes d)(e \otimes f) = (-)^{p(d)p(e)}\, ce \otimes df \tag{10}$$

where $p(d)$ and $p(e) \in \mathbb{Z}_2$ are the degrees (i.e. parities) of $d$ and $e$ respectively. On $\mathcal{F}^{\otimes n}$ the composition rules are, therefore, quite different. Let us denote

$$\begin{aligned} a_j &\equiv \mathbf{1} \otimes \mathbf{1} \otimes \ldots \otimes \mathbf{1} \otimes a \otimes \mathbf{1} \otimes \ldots \otimes \mathbf{1} \\ a_j{}^\dagger &\equiv \mathbf{1} \otimes \mathbf{1} \otimes \ldots \otimes \mathbf{1} \otimes a^\dagger \otimes \mathbf{1} \otimes \ldots \otimes \mathbf{1} \\ H_j &\equiv \mathbf{1} \otimes \mathbf{1} \otimes \ldots \otimes \mathbf{1} \otimes H \otimes \mathbf{1} \otimes \ldots \otimes \mathbf{1} \\ N_j &\equiv \mathbf{1} \otimes \mathbf{1} \otimes \ldots \otimes \mathbf{1} \otimes N \otimes \mathbf{1} \otimes \ldots \otimes \mathbf{1} \end{aligned}$$

where the multiple $\otimes$-products have $n$ factors in which the only element different from the identity $\mathbf{1}$ is in the $j$th position. One has for $(a, a^\dagger, N, \mathbf{1})$ in $h(1)$ the customary relations $[a_i, a_j] = 0$, $[a_i, a_j^\dagger] = \delta_{ij}\mathbf{1}$ and $[N_i, a_j] = -a_i\delta_{ij}$ (plus their Hermitian conjugates), while for $(a, a^\dagger, H)$ in $osp(1|2)$, the (graded) commutation relations are

$$\{a_i, {a_j}^\dagger\} = 2H_i\delta_{ij} \qquad [H_i, a_j] = -a_i\delta_{ij} \qquad [H_i, {a_j}^\dagger] = {a_i}^\dagger\delta_{ij}$$
$$\{{a_i}^\dagger, {a_j}^\dagger\} = \delta_{ij}J_i^+ \qquad \{a_i, a_j\} = \delta_{ij}J_i^-$$

and, of course, $[a_j, a_j] = 0$.

On the Fock basis of $\mathcal{F}^{\otimes n}$, adoption of $osp(1|2)$ leads to

$$\begin{aligned} {a_j}^\dagger|n_1, \ldots, n_{j-1}, n_j, \ldots, n_n\rangle &= (-1)^{s_j}\sqrt{n_j+1}|n_1, \ldots, n_{j-1}, n_j+1, \ldots, n_n\rangle \\ a_j|n_1, \ldots, n_{j-1}, n_j, \ldots, n_n\rangle &= (-1)^{s_j}\sqrt{n_j}|n_1, \ldots, n_{j-1}, n_j-1, \ldots, n_n\rangle \\ N_j|n_1, \ldots, n_{j-1}, n_j, \ldots, n_n\rangle &= n_j|n_1, \ldots, n_{j-1}, n_j, \ldots, n_n\rangle \end{aligned} \tag{11}$$

where the phases are exactly those customarily used for fermions [5] $\left(s_j \equiv \sum_{k=1}^{j-1} n_k\right)$.

It is worth noting that equations (11) differ from the usual (bosonic) equations only in the choice of phases and, on $\mathcal{F}^{\otimes n}$, imply $[a_i, {a_i}^\dagger] = \mathbf{1}$; this is consistent with the standard formulation, which in turn gives $\{a_i, {a_i}^\dagger\} = 2H_i (\equiv 2N_i + 1)$. Nevertheless, the subtle and important implication here is that in order to determine the phases of the basis vectors an order must be imposed *a priori* on the set of indices $j$ such that

$$|n_1, \ldots, n_j, \ldots, n_n\rangle \equiv \frac{1}{\sqrt{\prod_{j=1}^n n_j!}} ({a_1}^\dagger)^{n_1} \ldots ({a_j}^\dagger)^{n_j} \ldots ({a_n}^\dagger)^{n_n}|0\rangle$$

contrary to the standard bosonic theory where the creation operators commute and can be applied in any order. Of course, the usual properties of the Fock space, such as completeness:

$$\sum_{\{n\}} |n_1, n_2, \ldots\rangle\langle n_1, n_2, \ldots| = \mathbb{I}$$

and the projection operators on the one- and two-particles states

$$\mathcal{P}_1 \equiv \sum_i |1_i\rangle\langle 1_i| \qquad \mathcal{P}_2 \equiv \sum_{i<j} |1_i, 1_j\rangle\langle 1_i, 1_j| + \sum_i |2_i\rangle\langle 2_i| \tag{12}$$

where

$$\begin{aligned} |1_i\rangle &\equiv |n_1 = 0, n_2 = 0, \ldots, n_{i-1} = 0, n_i = 1, n_{i+1} = 0, \ldots, n_n = 0\rangle \\ |2_i\rangle &\equiv |n_1 = 0, n_2 = 0, \ldots, n_{i-1} = 0, n_i = 2, n_{i+1} = 0, \ldots, n_n = 0\rangle \\ |1_i, 1_j\rangle &\equiv |n_1 = 0, n_2 = 0, \ldots, n_i = 1, \ldots, n_j = 1, \ldots n_n = 0\rangle \end{aligned}$$

do not depend on such phases and persist. $osp(1|2)$ can be utilized in such a way to construct $n$-particle states, leading to a scheme that is non-equivalent to that derivable from $h(1)$ because of the grading of odd operators.

This possibility of using anticommuting operators without restriction on the occupation numbers $n_j$ also casts a new light on the question of how one should approach the introduction of statistics.

As stressed in the introduction, second quantization is essentially unrelated to statistics [6], which is required as a necessary set of rules to represent isomorphically $n$-particle states in the state-space given by the $n$-fold tensorization of the single-particle Hilbert space proper of first quantization. The relevant point in the analysis of this problem, performed by Pauli in [6], is that the symmetry with respect to the permutation of two particles does not depend on the prescription adopted to build $\mathcal{F}$ from the vacuum (i.e. on the commutation or anticommutation relations of the $a_i$'s and $a_j{}^\dagger$'s), but that it must be imposed as an external constraint aiming to guarantee a correct implementation of the above isomorphism.

In such a perspective let us consider what happens with two bosons. Independently of whether the algebra is graded or not, for such a system one has to consider a symmetric Hilbert space, that is a generic state vector must be symmetric with respect to the exchange of the two particles (this being the feature which qualifies them as boson):

$$|x_1, x_2\rangle_{\mathrm{B}} \equiv \tfrac{1}{\sqrt{2}}\left(|x_1\rangle|x_2\rangle + |x_2\rangle|x_1\rangle\right)$$

and by (12),

$$|x_1, x_2\rangle_{\mathrm{B}} = \sum_{i<j} |1_i, 1_j\rangle\langle 1_i, 1_j|x_1, x_2\rangle_{\mathrm{B}} + \sum_i |2_i\rangle\langle 2_i|x_1, x_2\rangle_{\mathrm{B}}.$$

Independently of how the states $|1_i, 1_j\rangle$ and $|2_i\rangle$ are constructed from the vacuum, the symmetry here is automatically implemented in that

$$\begin{aligned}\langle 1_i, 1_j|x_1, x_2\rangle_{\mathrm{B}} &= \tfrac{1}{\sqrt{2}}\langle 1_i, 1_j|\left(|x_1, x_2\rangle + |x_2, x_1\rangle\right) = \tfrac{1}{\sqrt{2}}\left(\langle 1_i|x_1\rangle\langle 1_j|x_2\rangle + \langle 1_i|x_2\rangle\langle 1_j|x_1\rangle\right)\\ \langle 2_i|x_1, x_2\rangle_{\mathrm{B}} &= \tfrac{1}{\sqrt{2}}\langle 2_i|\left(|x_1, x_2\rangle + |x_2, x_1\rangle\right) = \langle 1_i|x_1\rangle\langle 1_i|x_2\rangle\end{aligned} \tag{13}$$

are manifestly invariant with respect to the interchange of the two particles.

The feature that the statistics has no connection with the algebra is further proved by the fact that $osp(1|2)$, as well as $h(1)$, work equally well with fermions. For two fermions we must consider an antisymmetric Hilbert state-space

$$|x_1, x_2\rangle_{\mathrm{F}} \equiv \tfrac{1}{\sqrt{2}}\left(|x_1\rangle|x_2\rangle - |x_2\rangle|x_1\rangle\right).$$

Again independently of the algebra considered, the antisymmetry in the exchange of the two fermions is guaranteed, as well as the Pauli exclusion principle:

$$\begin{aligned}\langle 1_i, 1_j|x_1, x_2\rangle_{\mathrm{F}} &= \tfrac{1}{\sqrt{2}}\langle 1_i, 1_j|\left(|x_1, x_2\rangle - |x_2, x_1\rangle\right) = \tfrac{1}{\sqrt{2}}\left(\langle 1_i|x_1\rangle\langle 1_j|x_2\rangle - \langle 1_i|x_2\rangle\langle 1_j|x_1\rangle\right)\\ \langle 2_i|x_1, x_2\rangle_{\mathrm{F}} &= \tfrac{1}{\sqrt{2}}\langle 2_i|\left(|x_1, x_2\rangle - |x_2, x_1\rangle\right) = \tfrac{1}{\sqrt{2}}\left(\langle 1_i|x_1\rangle\langle 1_i|x_2\rangle - \langle 1_i|x_2\rangle\langle 1_i|x_1\rangle\right) = 0.\end{aligned} \tag{14}$$

Equations (13) and (14) clearly demonstrate the possibility—besides the customary scheme [5]—of constructing bosons with graded operators or fermions with even operators.

It should be stressed that our arguments in this letter are quite different from other procedures whereby *ad hoc* constraints are introduced on the variables in order to generate the statistics. An example of such different approaches is nonlinear transformation along the lines proposed by Gutzwiller's projection operator method [7]. In the fermionic case, a suggestive example is provided by the new creation and annihilation operators defined by

$$c_j = a_j Q_j \qquad Q_j \equiv \frac{1}{2}\frac{\left(1 - \mathrm{e}^{\mathrm{i}\pi N_j}\right)}{\sqrt{N_j + \frac{1}{2}\left(1 + \mathrm{e}^{\mathrm{i}\pi N_j}\right)}} = Q_j^\dagger \qquad c_j^\dagger = Q_j a_j^\dagger. \tag{15}$$

It is straightforward to check that—because of (2)—on the subsector of $\mathcal{F}^{\otimes n}$ consisting of paired superdoublets $\{|\ldots,2n_j,\ldots\rangle,|\ldots,2n_j+1,\ldots\rangle\}$ equation (15) leads to both the Pauli exclusion principle, $c_j^2=0=(c_j^\dagger)^2$, and the customary fermionic anticommutation relations $\{c_i,c_j{}^\dagger\}=\delta_{ij}\mathbf{1}$. The usual fermions can, therefore, be recovered by a restriction to $n_j=0$. Of course, in the above procedure no reference or use has been made of grading.

Adoption of a graded algebra also has an effect on the structures one can induce in the universal enveloping algebra (UEA). For example, it is usually assumed that the algebra $su(1,1)$ can be constructed in the UEA of $h(1)$. However, this is not possible because, as stressed before, $su(1,1)$ can easily be obtained from equations (3) as the bosonic sector of the superalgebra $osp(1|2)$, while one has to use $n_0=0$ (i.e. one also needs to impose indirectly the $osp(1|2)$ properties) to obtain the same result from equations (1). Moreover, the grading property (10) plays an essential role in obtaining the coalgebra (primitive of course) of $su(1,1)$ from the coalgebra of $S$, while it is impossible to obtain the same result from the coalgebra of $h(1)$.

Indeed, from (5), (8) and (9) we have, for instance, in $h(1)$, $\Delta(J^-)=a^2\otimes\mathbf{1}+\mathbf{1}\otimes a^2+2a\otimes a$ whereas in $osp(1|2)$, because of (11), $\Delta(J^-)=a^2\otimes\mathbf{1}+\mathbf{1}\otimes a^2\equiv J^-\otimes\mathbf{1}+\mathbf{1}\otimes J^-$, as it should because $J^-$ is primitive.

This shows that $su(1,1)$ is contained as a full Hopf algebra in the universal envelope of $S$, while only in the common representation (2) can the $su(1,1)$ algebra be considered realized in the UEA of $h(1)$ (all these can be extended to $su(2)$ by analytical continuation from $su(1,1)$).

We now recall that both $h(1)$ [8] and $osp(1|2)$ [9] have *quantum* deformations and the whole discussion can easily be extended to them. It should be kept in mind that, in the scheme proposed, there is no relation that links the algebraic features of the creation and annihilation operators to the symmetry of the states. It is, indeed, possible to study systems of particles, both fermions or bosons, by means of either $h_q(1)$ or $osp_q(1|2)$. The Fock space always remains the same, while differences appear in the relations of composed and single particle observables.

We finally conjecture that, in the present approach, there is room for considering objects with more complex symmetry such as anyons.

The authors gratefully acknowledge fruitful discussions with F Iachello.

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